The Extended Burr XII Distribution with Variable Shapes for the Hazard Rate

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Abstract

We define and study a new continuous distribution called the exponentiated Weibull Burr XII. Its density function can be expressed as a linear mixture of Burr XII. Its hazard rate is very flexibile in accomodating various shapes including constant, decreasing, increasing, J-shape, unimodal or bathtub shapes. Various of its structural properties are investigated including explicit expressions for the ordinary and incomplete moments, generating function, mean residual life, mean inactivity time and order statistics. We adopted the maximum likelihood method for estimating the model parameters. The flexibility of the new family is illustrated by means of a real data application.

Keywords: Burr XII Distribution, Exponentiated Weibull-H family, Generating Function, Maximum Likelihood.

1. Introduction

The statistical literature contains many generalized distributions which extensively used in modeling data in various applied areas such as reliability, insurance and life testing. The Burr-XII (BXII) distribution, proposed by Burr (1942), has many generalized forms in the literature such as the beta BXII distribution studied by Paranaíba et al. (2011), the Kumaraswamy BXII distribution introduced by Paranaíba et al. (2013), the beta exponentiated BXII distribution defined by Mead (2014), the Marshall-Olkin extended BXII distribution due to Al-Saiarie et al. (2014), the McDonald BXII distribution proposed by Gomes et al. (2015), the exponentiated Burr XII Poisson distribution due to da Silva et al. (2015), the Weibull BXII (WBXII) distribution studied by Afify et al.

(2017), the odd Lindley BXII distribution due to Abouelmagd et al. (2017) and the Kumaraswamy exponentiated BXII distribution defined by Mead and Afify (2017).

In this paper, we define and study a new five-parameter model named the *exponentiated Weibull Burr XII* (EWBXII) distribution which extends the WBXII distribution (Afify et al., 2017) and provide greater flexibility in modelling data. Based on the exponentiated Weibull-H (EW-H) family proposed by Cordeiro et al. (2017), we construct our newly model.

Consider the bsaeline cumulative distribution function (CDF) $H(x; \xi)$ with parameter vector ξ. Then, the CDF of the EW-H family is given by

$$F(x; v, a, b, \xi) = \left\{ 1 - \exp\left[-a \left(\frac{H(x;\xi)}{1-H(x;\xi)} \right)^b \right] \right\}^v. \tag{1}$$

The probability density function (PDF) of the EW-H family is

$$f(x; v, a, b, \xi) = abvh(x; \xi) \frac{H(x;\xi)^{b-1}}{[1-H(x;\xi)]^{b+1}} \exp\left[-a \left(\frac{H(x;\xi)}{1-H(x;\xi)} \right)^b \right]$$
$$\times \left\{ 1 - \exp\left[-a \left(\frac{H(x;\xi)}{1-H(x;\xi)} \right)^b \right] \right\}^{v-1}, \tag{2}$$

where v, a and b are three additional positive shape parameters which provide greater flexibility in accommodating all forms of the hazard rate function (HRF).

The EWBXII distribution is a flexible model which exhibit all forms of the hazard rate function (HRF) as shown in Figure 2. Further, the EWBXII distribution contains seventeen sub-models as special cases.

The rest of the paper is outlined as follows. In Section 2, we define the EWBXII distribution, present its special models and provide some plots of its PDF and HRF. In Section 3, we derive a linear representation for the EWBXII PDF and obtain some of its mathematical properties including ordinary and incomplete moments, mean residual life, mean inactivity time, moment generating function (mgf) and order statistics. In Section 4, we use maximum likelihood to estimate the model parameters. In Section 5, we use a real data set to prove empirically the flexibility of the new model. Finally, some concluding remarks are given in Section 6.

2 The EWBXII distribution

In this section, we define the EWBXII distribution, provide its special cases and give some plots for its PDF and HRF.
The CDF and PDF of the two parameter BXII distribution are given by (for $x > 0$)

$$H(x; \alpha, \beta) = 1 - (1 + x^\alpha)^{-\beta} \quad \text{and} \quad h(x; \alpha, \beta) = \alpha\beta x^{\alpha-1}(1 + x^\alpha)^{-\beta-1},$$

respectively, where α and β are positive shape parameters.

By inserting the CDF of the BXII in equation (1), we obtain the CDF of the EWBXII distribution

$$F(x) = \left(1 - \exp\left\{-a\left[(1+x^\alpha)^\beta - 1\right]^b\right\}\right)^v, x > 0$$

(3)

and the corresponding PDF of (3) is given by

$$f(x) = abv\alpha\beta x^{\alpha-1}\frac{\left[1-(1+x^\alpha)^{-\beta}\right]^{b-1}}{(1+x^\alpha)^{-\beta b+1}}\exp\left\{-a\left[(1+x^\alpha)^\beta - 1\right]^b\right\}$$
$$\times\left(1 - \exp\left\{-a\left[(1+x^\alpha)^\beta - 1\right]^b\right\}\right)^{v-1},$$

(4)

where α, β, a, b and v are positive shape parameters which can provide more flexibility to model various real world data.

Plots of the PDF and HRF of the EWBXII distribution are displayed in Figures 1 and 2, respectively. The plots in Figure 1 show that the PDF of the EWBXII distribution can be reversed J-shape, right-skewed, left-skewed, symmetric or concave down. Figure 2 shows that the HRF of the proposed model can be constant, decreasing, increasing, J-shape, unimodal, bathtub shapes.
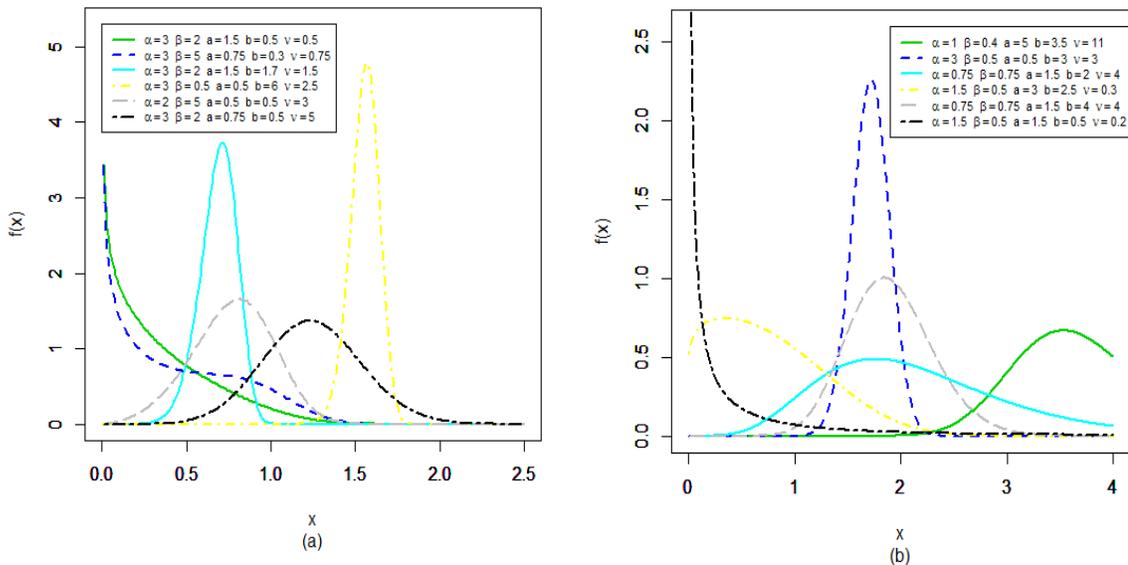
Figure 1: Some possible shapes for the EWBXII PDF
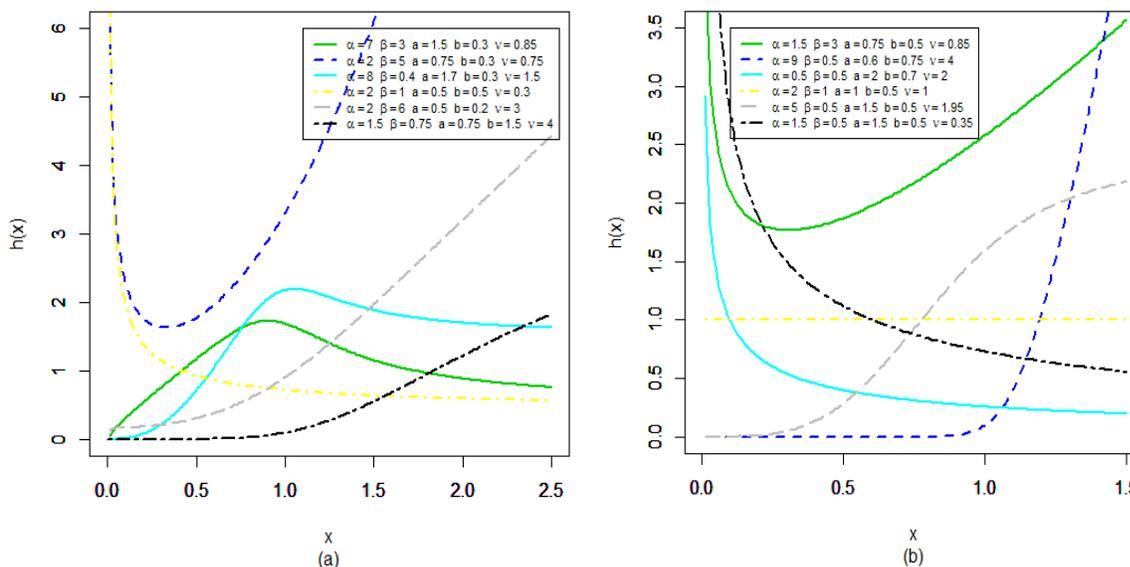
Figure 2: Some possible shapes for the EWBXII HRF

The EWBXII distribution is a comprehensive distribution which contains several lifetime models, such as Weibull BXII, exponential BXII (ExBXII), odd generalized exponential BXII (OGEx-BXII), Burr X BXII (BX-BXII), BXII, Lomax (Lo) and log logistic (LL) distributions as special cases. Table 1 provides all sub-models of the EWBXII distribution.

Table 1: Special cases of the EWBXII Distribution

No.	Model	α	β	a	b	ν	Author
1	WBXII	α	β	a	b	1	Afify et al. (2017)
2	W Lomax	1	β	a	b	1	-
3	W log-logistic	α	1	a	b	1	-
4	Ex BXII	α	β	a	1	1	-
5	Ex Lomax	1	β	a	1	1	-
6	Ex log-logistic	α	1	a	1	1	-
7	EW Lomax	1	β	a	b	ν	New
8	EW log-logistic	α	1	a	b	ν	New
9	OGEx-BXII	α	β	a	1	ν	New
10	OGEx-Lomax	1	β	a	1	ν	New
11	OGEx-log-logistic	α	1	a	1	ν	New
12	BX-BXII	α	β	1	2	ν	New
13	BX- Lomax	1	β	1	2	ν	New
14	BX- log-logistic	α	1	1	2	ν	New
15	BXII	α	β	1	1	1	Burr (1942)
16	Lomax	1	β	1	1	1	-
17	Log-logistic	α	1	1	1	1	-

3. Some EWBXII properties

Some properties of the EWBXII distribution including ordinary and incomplete moments, generating function, mean residual life (MRL), mean inactivity time (MIT) and order statistics are derived in this section.

3.1 Linear representation

According to equation (4.5) in Cordeiro et al. (2017), the PDF of the EWBXII distribution can be rewritten as

$$f(x) = \sum_{i,k,j=0}^{\infty} \frac{(-1)^{k+i} v b a^{k+1} (i+1)^k \Gamma(v) \Gamma([k+1]b+j+1)}{i!\, k!\, j!\, \Gamma(v-i) \Gamma([k+1]b+1)}$$
$$\times \alpha\beta x^{\alpha-1}(1+x^\alpha)^{-\beta-1}\left[1-(1+x^\alpha)^{-\beta}\right]^{(k+1)b+j-1}.$$

Applying the generalized binomial expansion to the last term, we have

$$f(x) = \sum_{m=0}^{\infty} \phi_m\, g_{\beta(m+1)}(x), \tag{5}$$

where the constant term is given by

$$\phi_m = \sum_{i,k,j=0}^{\infty} \frac{(-1)^{k+i+m} v b a^{k+1}(i+1)^k \Gamma(v)\Gamma([k+1]b+j)\Gamma([k+1]b+j+1)}{i!\, k!\, j!\, (m+1)!\, \Gamma(v-i)\Gamma([k+1]b+1)\Gamma([k+1]b+j-m)}$$

and $g_{\beta(m+1)}(x)$ refer to the BXII PDF with parameters α and $\beta(m+1)$. Equation (5) is important in deriving some properties of the EWBXII distributions from those of the BXII distribution.

3.2 Moments

The rth ordinary moment of X, follows from (5) (for $\alpha\beta(m+1) > r$) as

$$\mu'_r = \sum_{m=0}^{\infty} \phi_m\, \beta(m+1)\, B\left(\beta(m+1) - \frac{r}{\alpha}, \frac{r}{\alpha}+1\right).$$

The mean of X follows by setting $r = 1$ in the above equation.

The rth incomplete moment of the EWBXII distribution follows from (5) as

$$\varphi_r(t) = \sum_{m=0}^{\infty} \phi_m\, \beta(m+1)\, B\left(t^\alpha; \beta(m+1) - \frac{r}{\alpha}, \frac{r}{\alpha}+1\right).$$

When $r = 1$, we have the first incomplete moment of X

$$\varphi_1(t) = \sum_{m=0}^{\infty} \phi_m \, \beta(m+1) \, B\left(t^\alpha; \beta(m+1) - \frac{1}{\alpha}, \frac{1}{\alpha} + 1\right)$$

which is important to calculate the Bonferroni and Lorenz curves and the MRL and MIT.

The MRL or life expectancy at age t is defined by

$$m_X(t) = \frac{1 - \varphi_1(t)}{1 - F(t)} - t.$$

Using $\varphi_1(t)$, we obtain

$$m_X(t) = \frac{1}{1 - F(t)} \sum_{m=0}^{\infty} \phi_m \, \beta(m+1) \, B\left(t^\alpha; \beta(m+1) - \frac{1}{\alpha}, \frac{1}{\alpha} + 1\right) - t.$$

The MIT is defined (for $t > 0$) by

$$M_X(t) = t - \frac{\varphi_1(t)}{F(t)}$$

By inserting $\varphi_1(t)$ in the above equation, we have the MIT of X as

$$M_X(t) = t - \frac{1}{F(t)} \sum_{m=0}^{\infty} \phi_m \, \beta(m+1) \, B\left(t^\alpha; \beta(m+1) - \frac{1}{\alpha}, \frac{1}{\alpha} + 1\right).$$

The mgf of X follows from (5) as

$$M_X(t) = \sum_{m=0}^{\infty} \phi_m \, M_{\beta(m+1)}(t), \tag{6}$$

where $M_{\beta(m+1)}(t)$ is the mgf of the BXII distribution with two parameters α and $\beta(m+1)$. Paranaíba et al. (2011) provided a simple formula for the mgf of BXII distribution with two-parameter α and β (for $t < 0$) as

$$M(t) = s \, I\left(-t, \frac{s}{\beta} - 1, \frac{s}{\beta}, -\beta - 1\right). \tag{7}$$

Combining equations (6) and (7), the mgf of X reduces to

$$M_X(t) = s \sum_{m=0}^{\infty} \phi_m \, I\left(-t, \frac{s}{\beta(m+1)} - 1, \frac{s}{\beta(m+1)}, -\beta(m+1) - 1\right).$$

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3.3 Order statistics

Let $X_1, \ldots, X_n$ be a random sample of size n from the EWBXII distribution and let $X_{(1)}, \ldots, X_{(n)}$ be the corresponding order statistics. Then, the pdf of the ith order statistic, $X_{i:n}$, is given by

$$f_{i:n}(x) = \frac{n! f(x)}{(i-1)!(n-i)!} \sum_{j=0}^{n-i} (-1)^j \binom{n-i}{j} F(x)^{j+i-1}. \tag{8}$$

Based on equation (6.2) in Cordeiro et al. (2017), we have

$$f(x)F(x)^{j+i-1} = \sum_{l,k,m=0}^{\infty} \frac{(-1)^{l+k} vba^{k+1}(l+1)^k \Gamma([j+i]v)\Gamma([k+1]b+m+1)}{l!\, k!\, m!\, \Gamma([j+i]v-l)\Gamma([k+1]b+1)}$$
$$\times \alpha\beta x^{\alpha-1}(1+x^\alpha)^{-\beta-1}\left[1-(1+x^\alpha)^{-\beta}\right]^{(k+1)b+m-1}.$$

Using the generalized binomial expansion, the last equation reduces to

$$f(x)F(x)^{j+i-1} = \sum_{l,k,m,s=0}^{\infty} \frac{(-1)^{l+k+s} vba^{k+1}(l+1)^k \Gamma([j+i]v)\Gamma([k+1]b+m)}{l!\, k!\, m!\, s!\, \Gamma([j+i]v-l)\Gamma([k+1]b+1)}$$
$$\times \frac{\Gamma([k+1]b+m+1)}{\Gamma([k+1]b+m-s)} \alpha\beta x^{\alpha-1}(1+x^\alpha)^{-\beta(s+1)-1}.$$

Substituting the last equation in equation (8), the pdf of $X_{i:n}$ comes out as

$$f_{i:n}(x) = \sum_{s=0}^{\infty} t_s\, g_{\beta(s+1)}(x), \tag{9}$$

where

$$t_s = \frac{n!}{(i-1)!\,(n-i)!} \sum_{l,k,m,s=0}^{\infty} \sum_{j=0}^{n-i} \frac{(-1)^{j+l+k+s} vba^{k+1}(l+1)^k}{l!\, k!\, m!\, s!\,(s+1)\Gamma([j+i]v-l)} \binom{n-i}{j}$$
$$\times \frac{\Gamma([j+i]v)\Gamma([k+1]b+m)\Gamma([k+1]b+m+1)}{\Gamma([k+1]b+1)\Gamma([k+1]b+m-s)}$$

and $g_{\beta(s+1)}(x)$ as before is the BXII PDF with parameters α and $\beta(s+1)$.

Based on equation (9), the rth moment of $X_{i:n}$ reduces to

$$E(X_{i:n}^r) = \sum_{s=0}^{\infty} t_s\, \beta(s+1)\, B\left(\beta(s+1) - \frac{r}{\alpha}, \frac{r}{\alpha} + 1\right).$$

4. Estimation

In this section, the unknown parameters of the EWBXII model are estimated via the maximum likelihood from complete samples. Consider a random sample of size n,

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$x_1, \dots, x_n$, is drawn from the EWBXII distribution with parameter vector $\theta = (\alpha, \beta, a, b, v)^T$. Then, the log-likelihood for the EWBXII distribution comes out as

$$\ell = n\log(\alpha\beta abv) + (\alpha - 1)\sum_{i=1}^{n}\log x_i + (\beta b - 1)\sum_{i=1}^{n}\log s_i$$
$$+ (b - 1)\sum_{i=1}^{n}\log\left(1 - s_i^{-\beta}\right) - a\sum_{i=1}^{n}d_i^{\ b}$$
$$+ (v - 1)\sum_{i=1}^{n}\log\left[1 - \exp\left(-ad_i^{\ b}\right)\right],$$

where $s_i = 1 + x_i^{\alpha}$ and $d_i = s_i^{\beta} - 1$.

The score vector elements, $\boldsymbol{U}(\theta) = \frac{\partial \ell}{\partial \theta} = (U(\alpha), U(\beta), U(a), U(b), U(v))^T$, are given by

$$U(\alpha) = \frac{n}{\alpha} + \sum_{i=1}^{n}\log x_i - ab\sum_{i=1}^{n}\frac{t_i d_i^{\ b-1}}{s_i^{-2\beta}} + (\beta b - 1)\sum_{i=1}^{n}\frac{x_i^{\alpha}\log x_i}{s_i}$$
$$+ (b - 1)\sum_{i=1}^{n}\frac{t_i}{1 - s_i^{-\beta}} + ab(v - 1)\sum_{i=1}^{n}\frac{\frac{t_i d_i^{\ b-1}}{s_i^{2\beta}}\exp\left(-ad_i^{\ b}\right)}{1 - \exp\left(-ad_i^{\ b}\right)},$$

$$U(\beta) = \frac{n}{\beta} + b\sum_{i=1}^{n}\log s_i - ab\sum_{i=1}^{n}d_i^{\ b-1}s_i^{\beta}\log s_i - (b - 1)\sum_{i=1}^{n}\frac{s_i^{\beta}\log s_i}{1 - s_i^{-\beta}}$$
$$+ ab(v - 1)\sum_{i=1}^{n}\frac{d_i^{\ b-1}s_i^{\beta}\log s_i\exp\left(-ad_i^{\ b}\right)}{1 - \exp\left(-ad_i^{\ b}\right)},$$

$$U(a) = \frac{n}{a} - \sum_{i=1}^{n}d_i^{\ b} + (v - 1)\sum_{i=1}^{n}\frac{d_i^{\ b}\exp\left(-ad_i^{\ b}\right)}{1 - \exp\left(-ad_i^{\ b}\right)},$$

$$U(b) = \frac{n}{b} + \beta\sum_{i=1}^{n}\log s_i + \sum_{i=1}^{n}\log\left(1 - s_i^{-\beta}\right) - a\sum_{i=1}^{n}d_i^{\ b}\log d_i$$
$$+ (v - 1)\sum_{i=1}^{n}\frac{ad_i^{\ b}\exp\left(-ad_i^{\ b}\right)\log d_i}{1 - \exp\left(-ad_i^{\ b}\right)},$$

and

$$U(v) = \frac{n}{v} + \sum_{i=1}^{n}\log\left[1 - \exp\left(-ad_i^{\ b}\right)\right],$$

Respectively, where $t_i = \beta x_i^{\alpha}s_i^{-\beta-1}\log x_i$.

The estimate of the unknown parameters can be obtained by setting the score vector to zero, $\mathbf{U}(\hat{\theta}) = 0$. We can get the MLEs $\hat{\alpha}, \hat{\beta}, \hat{a}, \hat{b}$ and $\hat{v}$ by solving the above system of

equations simultaneously using numerically method with iterative techniques such as the Newton-Raphson algorithm.

5. Data analysis

In this section, we illustrate the flexibility and importance of the EWBXII distribution using a real data set. The data refer to nicotine measurements, made from several brands of cigarettes in 1998, collected by the Federal Trade. This data set contains $n = 346$ observations.

We shall compare the fit of the EWBXII distribution with some other competitive distributions, namely: the WBXII and beta BXII (Paranaíba et al., 2011), additive Weibull (AW) (Xie and Lai, 1995), beta Weibull (BW) (Lee et al., 2007), transmuted Weibull Lomax (TWL) (Afify et al., 2015), Kumaraswamy Marshall–Olkin Fréchet (KMOF) (Afify et al., 2016) and BXII distributions. The PDFs of these models are given by

WBXII: $f(x) = ab\alpha\beta x^{\alpha-1}\frac{[1-(1+x^\alpha)^{-\beta}]^{b-1}}{(1+x^\alpha)^{-\beta b+1}}\exp\left\{-a[(1+x^\alpha)^\beta - 1]^b\right\}, \alpha > 0, \beta > 0, a > 0$ and $b > 0$.

BBXII: $f(x) = \frac{c\theta\beta^{-c}}{B(a,b)}x^{c-1}\left[1+\left(\frac{x}{\beta}\right)^c\right]^{-\theta b-1}\left\{1-\left[1+\left(\frac{x}{\beta}\right)^c\right]^{-\theta}\right\}^{a-1}, \theta > 0, \beta > 0, a > 0, c > 0$ and $b > 0$.

AW: $f(x) = \left(\alpha\delta x^{\delta-1} + \gamma\beta x^{\beta-1}\right)e^{-\alpha x^\delta - \gamma x^\beta}, \alpha > 0, \beta > 0, \gamma > 0$ and $\delta > 0$.

BW: $f(x) = \frac{\beta\alpha^\beta}{B(a,b)}x^{\beta-1}e^{-b(\alpha x)^\beta}\left[1 - e^{-(\alpha x)^\beta}\right]^{a-1}, \alpha > 0, \beta > 0, a > 0$ and $b > 0$.

TWL: $f(x) = \frac{ab\alpha}{\beta}\left(1+\frac{x}{\beta}\right)^{b\alpha-1}\exp\left\{-a\left[\left(1+\frac{x}{\beta}\right)^\alpha - 1\right]^b\right\}$

$\times\left[1-\left(1+\frac{x}{\beta}\right)^{-\alpha}\right]^{b-1}\left\{1-\lambda+2\lambda e^{-a\left[\left(1+\frac{x}{\beta}\right)^\alpha-1\right]^b}\right\}, \qquad \alpha > 0, \beta > 0, a > 0, b > 0$ and $|\lambda| \le 1$.

KMOF: $f(x) = \alpha\beta ab\delta^\beta x^{-\beta-1}\exp\left[-a\left(\frac{\delta}{x}\right)^\beta\right]\left\{\alpha + (1-\alpha)\exp\left[-\left(\frac{\delta}{x}\right)^\beta\right]\right\}^{-a-1}$

$\times\left\{1 - \exp\left[-a\left(\frac{\delta}{x}\right)^\beta\right]\left\{\alpha + (1-\alpha)\exp\left[-\left(\frac{\delta}{x}\right)^\beta\right]\right\}^{-a}\right\}^{b-1}, \alpha > 0, \beta > 0, a > 0, b > 0$ and $\delta > 0$.

The Akaike information criterion (AIC), consistent Akaike information criterion ($CAIC$), Bayesian information criterion (BIC), Hannan-Quinn information criterion ($HQIC$) and $-\hat{\ell}$, where $\hat{\ell}$ is the maximized log-likelihood, are used for comparing the fitted models.

Tables 2 and 4 list the values of $-\hat{\ell}, AIC, CAIC, BIC$ and $HQIC$ whereas the MLEs and their corresponding standard errors (in parentheses) of the model parameters are given in Tables 3 and 5.

Table 2: Goodness-of-fit statistics

Model	$-\hat{\ell}$	AIC	CAIC	BIC	HQIC
EWBXII	106.709	223.419	223.595	242.651	231.077
BBXII	107.856	225.712	225.888	244.945	233.371
WBXII	112.289	232.579	232.697	247.965	238.706
BW	112.587	233.173	233.29	248.559	239.300
AW	113.291	234.581	234.698	249.966	240.707
TWL	112.916	235.832	236.009	255.064	243.491
KMOF	114.034	238.068	238.244	257.299	245.726
BXII	127.294	258.587	258.622	266.280	261.651

Table 3: MLEs and their SEs (in parentheses)

Model	Estimates				
EWBXII (α,β,a,b,ν)	2.0186 (0.3822)	11.4275 (3.3495)	2.0765 (0.9761)	0.0869 (0.0425)	24.1274 (23.1607)
BBXII (a,b,c,β,k)	0.2086 (0.0148)	0.7285 (0.2535)	10.4641 (0.1915)	1.1101 (0.0483)	0.9449 (0.3098)
WBXII (α,β,a,b)	0.7219 (0.2658)	2.1971 (0.8762)	0.0339 (0.0527)	2.7275 (1.3047)	
BW (α,β,a,b)	0.6686 (0.578)	3.1645 (0.426)	0.7784 (0.163)	3.0922 (8.174)	
AW $(\alpha,\beta,\gamma,\delta)$	1.135 (0.062)	0.3084 (0.1)	0.0002 (1.369×10^{-3})	2.7219 (0.114)	
TWL $(\alpha,\beta,a,b,\lambda)$	7.8903 (15.497)	17.1065 (44.562)	3.8175 (4.1843)	2.3499 (0.309)	0.1600 (0.214)
KMOF $(\alpha,\beta,\delta,a,b)$	20410.35 (137.2224)	1.1973 (0.0109)	0.0109 (0.0009)	2.2892 (0.0667)	36699.16 (12799.87)
BXII (α,β)	3.4865 (0.1539)	1.8087 (0.0973)			

In Table 2, we compare the fits of the EWBXII distribution with the BBXII, WBXII, BW, AW, TWL, KMOF and BXII distributions. One can see, from Table 1, that the EWBXII distribution has the lowest values for goodness-of-fit statistics among all fitted models. So it could be chosen as the best model for the nicotine data.

The histogram and the estimated densities for nicotine data are displayed in Figure 3. These plots reveal that the EWBXII distribution is the best model to fit this data set. The fitted PDF, CDF, survival function (SF) and Q-Q plots of the EWBXII distribution are shown in Figure 4.
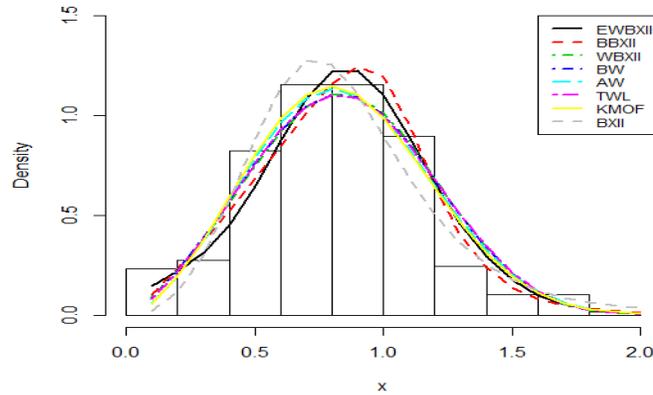
Figure 4: Fitted PDF of the EWBXII distribution and other fitted PDFs
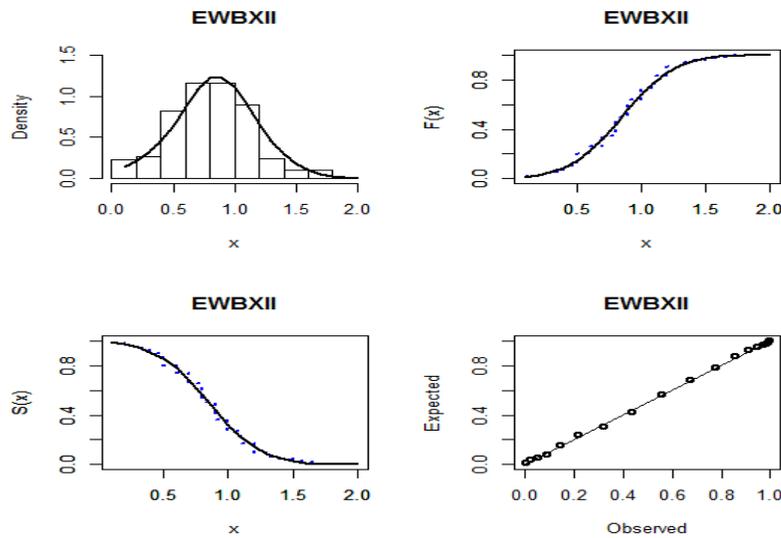
Figure 5: Fitted PDF, CDF, SFand Q-Q plots of the EWBXII distribution

6. Conclusions

We study a new five-parameter model called the exponentiated Weibull Burr XII (EWBXII) distribution which generalizes the two-parameter Burr XII distribution and …. We provide some mathematical properties of the new family including explicit expansions for the ordinary and incomplete moments, quantile and generating functions and entropies. The maximum likelihood estimation of the model parameters is investigated. By means of a real data application, we show that the EWBXII distribution can provide better fit than some other well-known models.

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